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Dielectric Barrier Discharge for Aerodynamic Flow Control

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ABSTRACT

Dielectric barrier discharge (DBD) plasma actuators have been used worldwide for aerodynamic flow-control. It is known that the effect of DBD is to produce a local body force within the boundary layer, which produces a wall jet. An analytical work has been carried out using the classical triple deck theory for subsonic flows to understand the flow dynamics with the local body force within the boundary layer. The results predict suction upstream of the disturbance (body force) and pumping downstream, which are in qualitative agreement with experiments conducted at LHSR, IISc.

Key Words: DBD, Plasma, Triple Deck theory, Body force, flow control, Wall shear stress

1. Introduction

Dielectric barrier discharge (DBD) plasma actuators have been used worldwide for aerodynamic flow-control application spreading across the speed regimes [1,2,3,4]. These actuators are being proposed to reduce the separated flow regions in the flow field and are generally perceived to add a finite amount of momentum to the flow within the boundary layer. However the precise flow dynamics associated with the working of these actuators still remains an open question.

A typical DBD configuration consists of two electrodes, one uncoated and exposed to air and the other encapsulated by a dielectric material. The electrodes are arranged in a highly asymmetric geometry. These electrodes when supplied with an AC voltage at high enough levels (1 to 50kV) and frequency (1 to 10 kHz) causes the air over the covered electrode to weakly ionize and results in plasma formation. The plasma formed consists of ions and electrons on which acts the imposed electric field. This results in the acceleration of the ions in the direction of electric field (electrons due to their low mass are neglected). These ions now collide with neutral fluid molecules and results in a body force (fig.1). The body force is the mechanism for active aerodynamic flow control.

Experiments in quiescent air has shown that the body force generated leads to a wall jet whose peak velocity can reach as high as 10m/s[2]. Previous smoke flow visualization experiments conducted in LHSR also confirmed the existence of a near wall tangential wall jet. The experimental setup used and plasma produced are shown in figs 2 and 3 while figs.4 and 5 indicate entrainment of ambient air from surroundings due to the wall jet.

Although several numerical works have appeared related to DBD [5,6] ,no study has been done so far to understand the fundamental physics due to the presence of a local body force within the boundary layer. In the present work, we analytically investigate the effect of a local body force along the flow direction within the boundary layer for high Reynolds number subsonic flows. First order triple deck theory [7] is used for this. Thus surface pressure and wall shear variations are obtained.

2. Methodology

Triple deck theory is used in the present study, whose structure is shown in Fig.6, with a generalized body force within the lower deck. All the classical subsonic triple deck equations remain the same except for the lower deck equations, where a body force term is added on the pressure gradient side. The upper and main deck equations are then linear and are thus solved in closed form. The lower deck equation is non linear. It is linearised with Blasius profile using small body force assumption and is then solved. The obtained results predict the source of wall jet.

3. Mathematical Formulation

The 2-D continuity and momentum equations on usual scaling reduces to

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{R}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{R}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

where $R=\frac{L^* U_\infty^*}{\nu^*}$ is the flow Reynolds number with L^* the location of disturbance.

3.1 Main Deck

The asymptotic series for the main deck are

$$u=u_0\left(y^{(m)}\right)+\varepsilon u_1^{(m)}\left(x,y^{(m)}\right)+\ldots$$
$$v=\varepsilon^2 v_1^{(m)}\left(x,y^{(m)}\right)+\ldots$$
$$p=\varepsilon^2 p_1^{(m)}\left(x,y^{(m)}\right)+\ldots$$

which results in the equations

$$\frac{\partial u_1^{(m)}}{\partial x}+\frac{\partial v_1^{(m)}}{\partial y}=0,$$
$$u_0\frac{\partial u_1^{(m)}}{\partial x}+v_1^{(m)}\frac{du_0}{dy^{(m)}}=0,$$
$$and\ \frac{\partial p_1^{(m)}}{\partial y^{(m)}}=0$$

Solving gives

$$p_1^{(m)}=p_1^{(m)}(x)$$

$$v_1^{(m)}\left(x,y^{(m)}\right)=u_0\left(y^{(m)}\right)A'(x),$$

$$u_1^{(m)}=u_0'\left(y^{(m)}\right)A(x)$$

$$where\ A(x)\ is\ the\ displacement\ function$$
$$and\ \lim_{x\to-\infty}A(x)=0$$

3.2 Upper deck

The asymptotic series for the upper deck are

$$u=1+\varepsilon^2 u_1^{(u)}\left(x,y^{(u)}\right)+\ldots$$
$$v=\varepsilon^2 v_1^{(u)}\left(x,y^{(u)}\right)+\ldots$$
$$p=\varepsilon^2 p_1^{(u)}\left(x,y^{(u)}\right)+\ldots$$

and the resulting equations are

$$\frac{\partial u_1^{(u)}}{\partial x}+\frac{\partial v_1^{(u)}}{\partial y}=0,$$
$$\frac{\partial u_1^{(u)}}{\partial x}=\frac{\partial p_1^{(u)}}{\partial x},$$
$$\frac{\partial v_1^{(u)}}{\partial x}=\frac{\partial p_1^{(u)}}{\partial y^{(u)}},$$

which on solving gives

$$u_1^{(u)}\left(x,y^{(u)}\right)=p_1^{(u)}\left(x,y^{(u)}\right)$$

$$=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{u_1^{(u)}(\xi,0)\,y^{(u)}}{(\xi-x)^2+y^{(u)2}}d\xi\ and$$

$$v_1^{(u)}\left(x,y^{(u)}\right)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{(\xi-x)u_1^{(u)}(\xi,0)}{(\xi-x)^2+y^{(u)2}}d\xi$$

and these relations provide the classical pressure displacement law

$$\frac{dA}{dx}=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{p_1^{(u)}(\xi,0)}{(\xi-x)}d\xi$$

3.3 Lower Deck

The asymptotic series are

$$u=\varepsilon u_1^{(L)}\left(x,y^{(L)}\right)+0\left(\varepsilon^3\right)+\ldots$$
$$v=\varepsilon^3 v_1^{(L)}\left(x,y^{(L)}\right)+0\left(\varepsilon^4\right)+\ldots$$
$$p=\varepsilon^2 p_1^{(L)}\left(x,y^{(L)}\right)+\ldots$$

which (after some more scalings) leads to

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}+kf\left(x\right)$$

and $\frac{\partial p}{\partial y}=0$

$$where\, kf\,(x)\, is\, the\, body\;\; force\, and\; k\in(0,1]$$
$$the\, boundary\, conditions\, being$$
$$u=v=0\;\; at\;\; y=0,$$
$$u\rightarrow\left(y+A\left(x\right)\right)\; as\; y\rightarrow\infty$$
$$(matching\; with\, main\, deck)$$
$$u\rightarrow y\;\; as\;\; x\rightarrow-\infty$$
$$(matching\;\; with\, Blasius)$$

(superscripts have been dropped for brevity.)

As seen from above, the lower deck equations are non-linear and hence difficult to solve in closed form. Further progress is made by assuming that the body force is small (i.e. k<<1). This implies that the resulting flow field is just a correction to the oncoming Blasius profile. Hence we demand

$$u=y+k\bar{u},$$
$$v=k\bar{v},$$
$$p=k\bar{p}$$
$$A=k\bar{A}$$

which leads to (on dropping all upper ‘ – ‘ s)

$$y\frac{\partial u}{\partial x}+v=-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}+f\left(x\right)$$

Differentiating with respect to y gives

$$y\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial^3 u}{\partial y^3}$$

Taking Fourier transform (defined as $F\left(\omega\right)=\int_{-\infty}^{\infty}e^{-i\omega x}f(x)dx$)

gives Airy’s differential equation

$$\frac{d^3U}{dy^3}=i\omega y\frac{dU}{dy}$$

which on solving gives

$$U(\omega,y)=c\left(\omega\right)\int_0^y Ai\left[\left(i\omega\right)^{\frac{1}{3}}y\right]dt$$

Similarly carrying out Fourier transform of the pressure-displacement law gives

$$p\left(\omega\right)=-i\left(i\omega\right)\mathrm{sgn}\left(\omega\right)A\left(\omega\right)$$

Also at the wall (y=0)

$$\left(i\omega\right)p=F+CA_i^{'}\left(0\right)(iw)^{1/3}$$

Solving above equations leads to

$$P=\frac{(i\omega)^{\frac{1}{3}}}{D}F$$

$$C=3i(i\omega)^{-\frac{1}{3}}\mathrm{sgn}(\omega)\frac{F}{D}$$

$$A=i(i\omega)^{-2}\mathrm{sgn}(\omega)\frac{F}{D}$$

$$where\;\; D=(i\omega)^{(4/3)}+i\vartheta^{(4/3)}sgn(\omega),\;\vartheta=.8272\,,$$

Inverse transforms of all these are obtained by introducing a branch cut along the entire imaginary axis in the ω-plane and then integrating along specific paths. These provide

$$P\left(x\right)=\frac{k}{2\pi}\int_{-\infty}^{\infty}\alpha\left(t\right)f\left(x-t\right)dt$$

$$where\quad \alpha\left(t\right)=\begin{cases}\displaystyle\int_0^\infty\frac{s^{\frac{1}{3}}e^{-(\theta t)s}}{s^{\frac{8}{3}}-\sqrt{3}s^{\frac{4}{3}}+1}ds, & t>0\\ \displaystyle-2\int_0^\infty\frac{s^{\frac{1}{3}}e^{(\theta t)s}}{s^{\frac{8}{3}}+1}ds, & t<0\end{cases}$$

and

$$\frac{\tau_0-1}{k}=\frac{3}{2\pi}A_i^{'}\left(0\right)\theta^{-\frac{2}{3}}\int_{-\infty}^{\infty}\beta\left(t\right)f\left(x-t\right)dt$$

$$\beta(t)=\begin{cases}\displaystyle\int_0^\infty \frac{(\sqrt{3}s^{-\frac{1}{3}}-s)e^{-(\theta t)s}}{s^{\frac{8}{3}}-\sqrt{3}s^{\frac{4}{3}}+1}ds, & t>0\\[2ex] \displaystyle 2\int_0^\infty \frac{se^{(\theta t)s}}{s^{\frac{8}{3}}+1}ds, & t<0\end{cases}$$

4. Results and Discussion

As seen from above, $\alpha(t)<0$ for t<0 and $\alpha(t)>0$ for t>0. This means for any positive body force distribution (i.e. force directed along the flow) pressure drops below ambient for x<0 and rises for x>0. This is confirmed by assuming a general body force distribution,

$$f(x)=1/(1+x^2)$$

The variations of pressure and wall shear are as shown in figs. 7 and 8. Clearly suction is present upstream of the body force and pumping is present downstream (fig.7). The suction effect draws in fluid from $x\to-\infty$ and the blowing effect pushes this sucked in fluid to $x\to\infty$. It is this combined action of suction and blowing that leads to the well known ‘wall jet.’ Thus the presence of a local body force is equivalent to having a ‘mechanical pump’ at x=0 with suction and discharge sides located at x<0 and x>0 respectively.
Another important observation is that pressure gradient is favorable (dp/dx<0) throughout the region where body force effect is present (except for a small region near the origin). As a result wall shear stress increases beyond the Blasius value and leads to well attached flow (fig 8). This is the fundamental mechanism for delaying any flow separation tendencies and thus justifies their use in aerodynamic flow control.

4. Conclusion

A successful analytical work has been carried out to understand the flow dynamics of flat plate boundary layer with local concentrated body force. This finds application in DBD used for aerodynamic flow control. The analytical results predict suction upstream and pumping downstream which deciphers the source of wall jet observed experimentally.

5. Acknowledgements

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6. References

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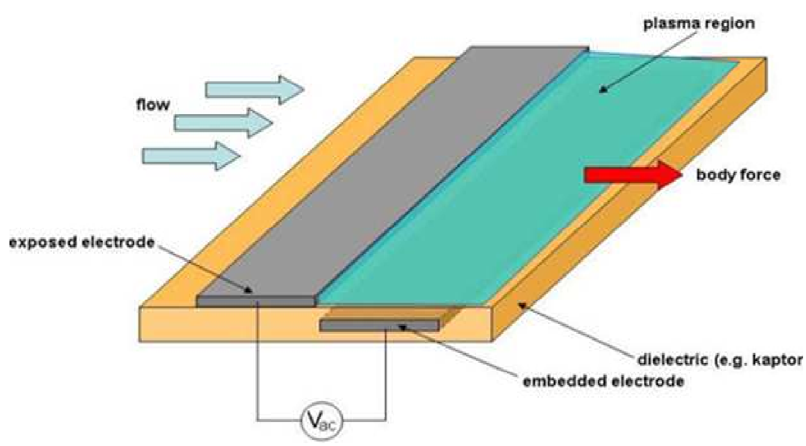


Fig.1 Typical DBD configuration

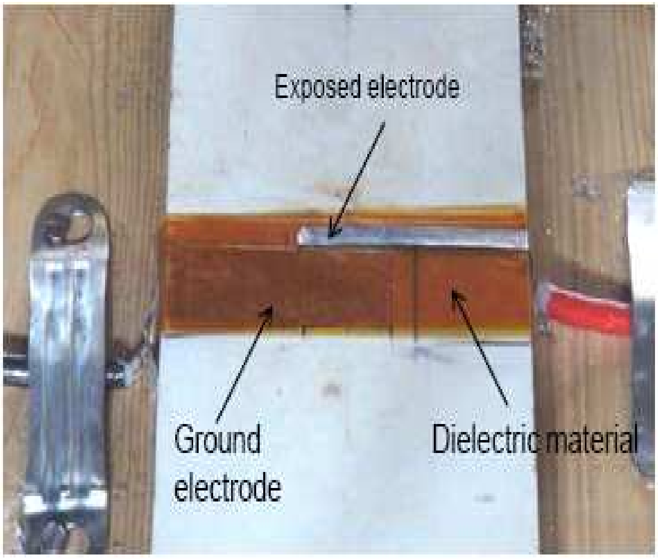


Fig.2 Experimental setup [12]

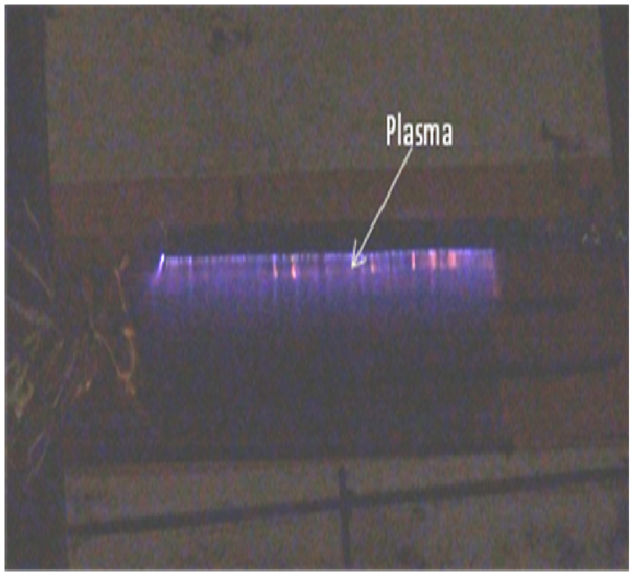
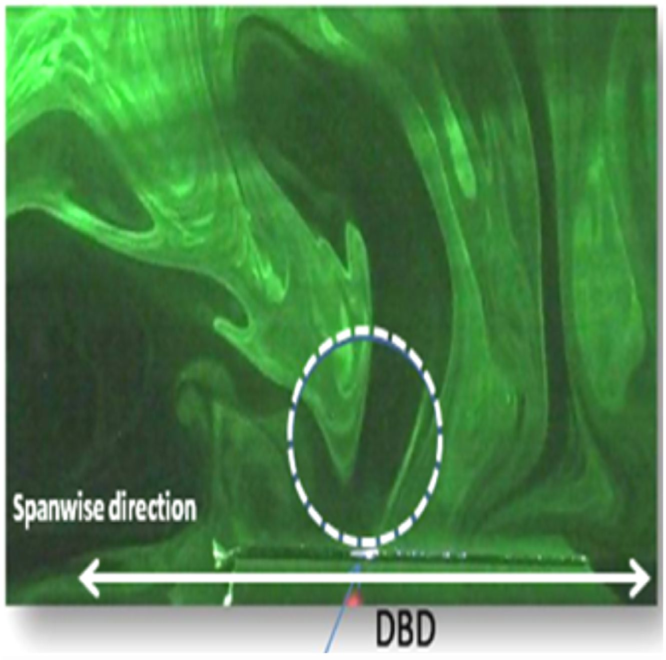


Fig.3 Plasma produced by DBD [12]



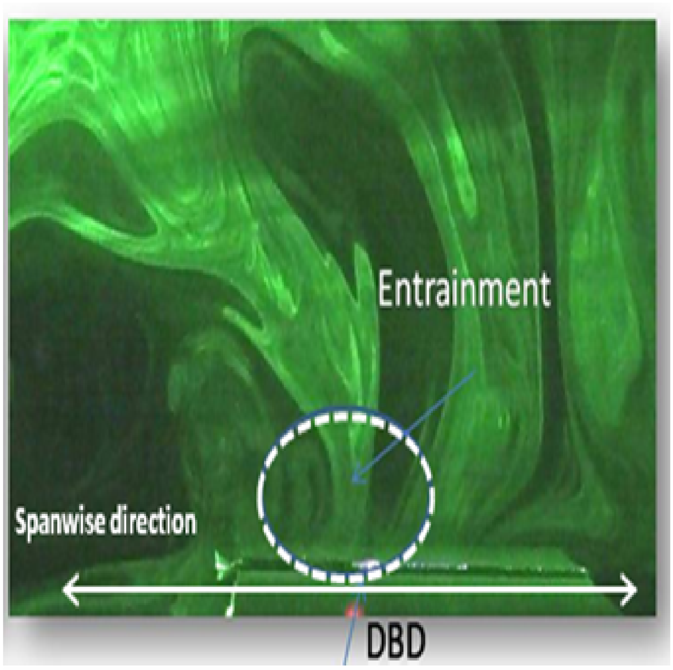


Fig.4 [12] **Fig.5 [12]**

Suction and pumping observed by smoke flow visualization

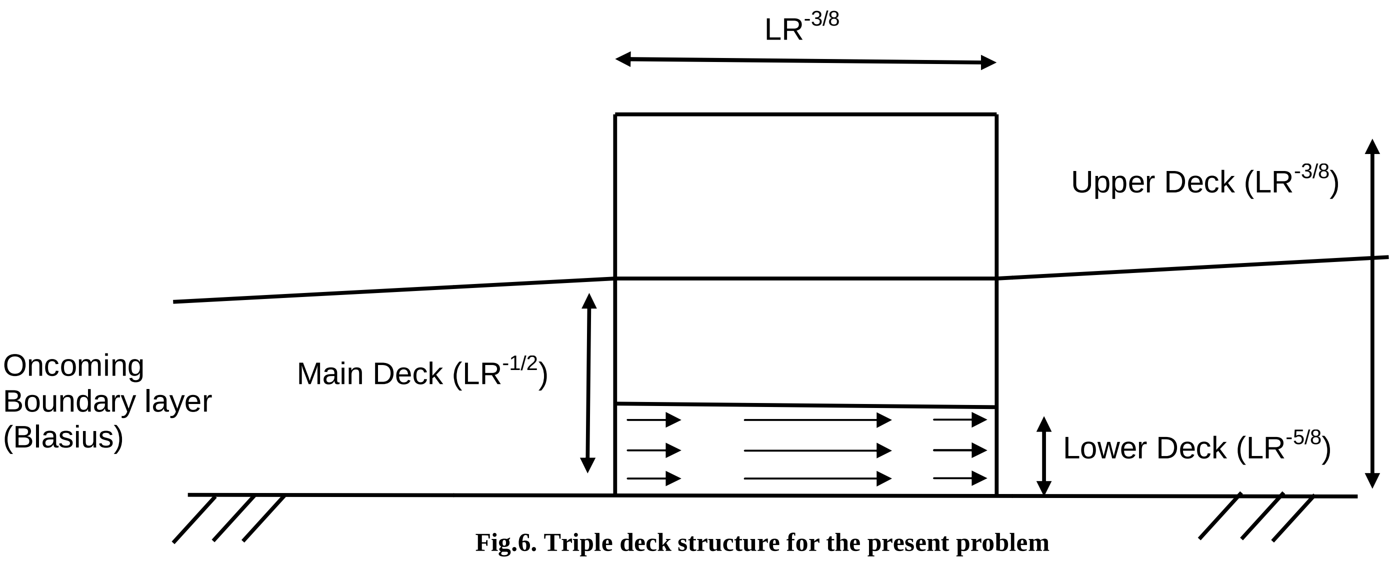


Fig.6. Triple deck structure for the present problem

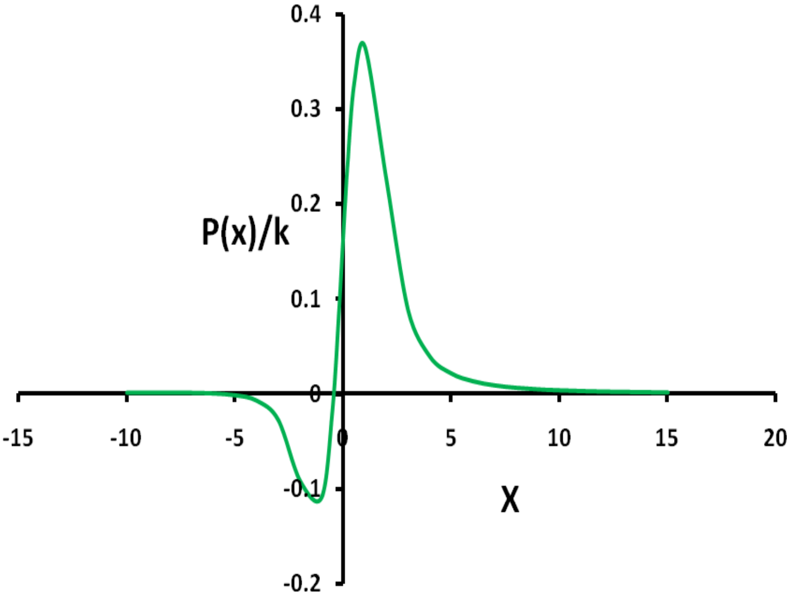


Fig.7 Pressure variation due to the body force

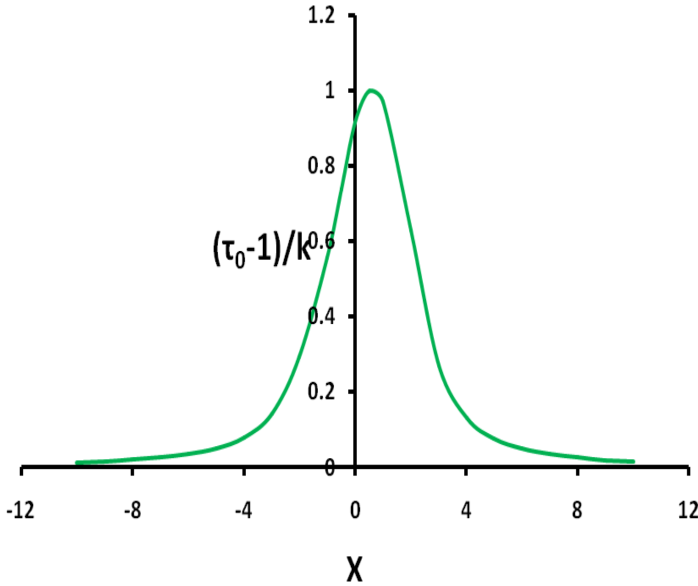


Fig.8 Wall shear variation due to the body force